Abstract—The theory of symmetrical components and the synthesis of sequence networks for three-phase power systems are instrumental for solving most unbalanced problems such as asymmetrical faults, as well as for understanding the unbalanced operating conditions of a normally balanced system and the behavior and influence of harmonic voltages and currents.

Unfortunately, the theory of symmetrical components is often learned merely as a set of abstract algebraic equations without any practical insight. The novel approach for understanding symmetrical components and synthesizing sequence networks presented in this manuscript enlightens the engineer to the reasons behind the behaviors observed on an unbalanced three-phase power system. The proposed approach also helps to reduce commonly-made errors in unbalanced system calculations.

Index Terms—Fault currents, power engineering education, sequence networks, symmetrical components

I. INTRODUCTION

Any power system calculations involve analysis of a balanced three-phase system, for which only one phase needs to be analyzed. The symmetry of the problem allows the behavior of the other two phases to be determined based on the calculated behavior of the first phase. This single-phase equivalent approach greatly simplifies the calculation process.

But when the system entails an unbalanced system of voltage and current phasors, the single-phase analysis approach cannot be directly applied. This situation is encountered in determining the system response to an unbalanced short-circuit fault. The option of analyzing the unbalanced system as a three-phase problem is not appealing, since the resulting mathematics would be cumbersome and very difficult to solve. Use of a single-phase approach would be possible if the unbalanced phasors could be resolved into balanced components. Charles Fortescue demonstrated in 1918 that resolving an unbalanced set of voltage or current phasors into a set of balanced components is always possible\(^1\). Before developing the symmetrical components of an unbalanced set of three-phase phasors, we look at a more straightforward example of resolving a vector into components.

II. COMPREHENDING VECTOR COMPONENTS

A. Physical Example of Vector Components

A basic problem in statics involves calculating the reaction at the attachment point of a cantilevered beam when subjected to a vertical loading, as shown in Fig. 1(a). The downward force \(F\) tries to produce a clockwise rotation of the beam about the attachment point with the wall called a moment. The moment produced by force \(F\) is the product of the force magnitude and the perpendicular distance from the force to the attachment point, or \(M = F \times d\).

\[\text{Figure 1 – Loadings on Cantilevered Beam}\]

A slight modification to this problem leads to a complication. Fig. 1(b) shows another force \(F\) applied to the same beam, but this time, \(F\) is not directed downward, but at an angle \(\theta\) with respect to the vertical.

\[\text{Figure 2 – Force } F \text{ Resolved into Appropriate Components}\]

Now, the resulting moment at the attachment point is not \(F \times d\), since \(F\) and \(d\) are not orthogonal. In order to calculate the moment, we can resolve the force \(F\) into two components: one perpendicular to the \(d\) vector (the component we need to calculate the moment), and one parallel to the \(d\) vector (which produces no moment). The resolution of \(F\) into these components is shown in Fig. 2.
moment equation (\(F_V\) is perpendicular to \(d\), thus determining the moment, and \(F_H\) is parallel to \(d\), which results in no moment).

Trigonometry can be used to express \(F_V\) and \(F_H\) in terms of \(F\) and \(\theta\):
\[
F_V = F \cos \theta \quad \text{and} \quad F_H = F \sin \theta. \tag{1 \& 2}
\]

Equations (1) and (2) allow transformation from the original parameters \(F\) and \(\theta\) into a component environment to facilitate calculation of the moment. A constraint which must be enforced is that the vector sum of \(F_V\) and \(F_H\) equals the original force \(F\).
\[
F = F_V + F_H \tag{3}
\]

The angle \(\theta\) can also be represented in terms of \(F_V\) and \(F_H\) as
\[
\theta = \tan^{-1} \left( \frac{F_H}{F_V} \right). \tag{4}
\]

Equations (3) and (4) allow transformation from the component environment back to the original parameters of \(F\) and \(\theta\).

Hence, starting with the \(F\) and \(\theta\) environment, we can convert to the component environment of \(F_V\) and \(F_H\) to facilitate calculation of the moment. This same concept can be applied to developing a system of components to facilitate the calculation of unbalanced voltages and currents on a normally-balanced three-phase system. If the location of the derived neutral point does not coincide with the balanced neutral point, such as with a four-wire delta system, the system is inherently unbalanced. In such a system, this method of applying symmetrical components cannot be used\(^2\).

Since the current phasors are unbalanced, meaning that each may have a different magnitude and different angular separation from the other two phasors, we cannot analyze the system taking a single-phase equivalent approach unless we first resolve the unbalanced phasors into a suitable set of balanced components.

Since the three-phase system of current phasors has more degrees of freedom than the beam example, we need not two but three sets of components to represent the unbalanced phasors. The necessary requirements for the symmetrical components of the unbalanced currents are 1) the magnitudes of each of the phasors of a given set of components are equal, and 2) the angular separation between any two phasors in a given set of components is equal.

Before determining the symmetrical components of the unbalanced phasors, we need to understand the concept of phase sequencing. Quite often, phase sequencing is referred to as phase rotation, but this terminology is very misleading and is technically incorrect. In fact, all phasors rotate counterclockwise—always. While the direction of rotation never changes, the sequencing of the phasors may change.

Referring to Fig. 5(a), the observer is looking directly at the phase A phasor. The next phasor to come around and point at the observer is the one from phase B. Finally, the phase C phasor will rotate by the observer. This sequencing defines an A-B-C, or positive sequence, set of components.

In contrast, Fig. 5(b) shows an example of an A-C-B, or negative sequence, set of components.

Now we can define the three sets of balanced components to represent the unbalanced set of current phasors shown in Fig. 4. The first set will be a balanced set of phasors having the same phase sequencing as the unbalanced currents. We will call this set the positive sequence components, and denote the positive sequence values with the subscript 1.

The second set of components will be a balanced set of phasors having A-C-B phase sequencing. We will call this set the negative sequence components, and will denote these phasors with the subscript 2.

For the third set of components, we will choose a set with equal magnitudes, and no angular displacement between the phases. Note that an angular separation of zero also fulfills the definition of balanced. This set of components is the zero sequence, and we use the subscript 0 to denote them.

Studying the sequence components shown in Fig. 6 enables us to see that we can exploit the symmetry of the systems to simplify the nomenclature. We can define an operator \(a\) such that multiplying any phasor by a simply rotates the original phasor by 120°. Thinking in terms of polar coordinates, it is evident that the \(a\) operator must have a magnitude of 1, or multiplying a phasor by \(a\) would rescale the phasor.
To achieve the 120° rotation, the angle of the $a$ operator must be 120°, since angles are additive when multiplying numbers in polar form. Therefore,

$$a = \frac{1}{120°}.$$  (5)

The $a$ operator can also be expressed in rectangular form as

$$a = \frac{1}{2} + j \frac{\sqrt{3}}{2},$$  (6)

where $j = \sqrt{-1}$. Just as $a$ can be thought of as a 120° rotator, $j$ can be viewed as a 90° rotator. Visualizing the effect of $a$ and $j$ is important, since doing so makes the problem less mathematical and more graphic.

Using the $a$ operator, we can eliminate the double subscript notation used in Fig. 6 by expressing each phasor in terms of the phase A phasor. This process brings us to a single-phase equivalent of the original system – the goal we were attempting to attain.

The symmetrical components must satisfy the constraint that their vector sum equals the original set of unbalanced phasors. This relationship is shown graphically in Fig. 8.

Using (7) through (12) as transformation equations, we can apply the theory of symmetrical components to unbalanced conditions on otherwise balanced three-phase power systems. But before we do, we must understand the electrical characteristics of the sequence currents so we can understand how they behave in the three-phase system. This knowledge is essential for the proper synthesis of the sequence networks.

### III. UNDERSTANDING SEQUENCE COMPONENTS

#### A. Electrical Characteristics of the Sequence Currents

Fig. 9(a) shows a wye-connected source supplying an unspecified load, which is required to ensure a closed path for current flow.

The top wire is carrying a current $I$ from the source to the load, we know a return path back to the source must exist. When there is an angular displacement between the line currents, the middle and bottom wires serve as the return path for the source current flowing in the top wire. Writing a node equation using the load as the node, we see that $I = x + y$. This relationship can be verified graphically by drawing the three line currents in the time domain, shown in Fig. 9(b). At a specific time $t = T$, the instantaneous values of the currents in the three wires sum to zero, thus honoring Kirchhoff’s Current Law. Thus, for the positive and negative sequence, a current supplied by one phase conductor is returned to the source by the other two. This relationship is always true in three-phase circuits.
The zero sequence current, however, behaves differently. There is no angular displacement between the phases, so whatever instantaneous current flows on the top wire also must flow on the middle and bottom wires.

Fig. 10(a) shows a total of $3I_0$ delivered from the source to the load. A return path to the source must exist. The zero sequence current is supplied to the load on the phase conductors, but it cannot return to the source on the phase conductors. A fourth conductor must be present to serve as the return path. The neutral conductor returns the zero-sequence current supplied by each phase conductor, or $3I_0$, as shown in Fig. 10(b). If a fourth conductor (return path) does not exist, zero-sequence current will not flow. This property is a characteristic of three-phase circuits.

![Figure 10 – Zero-Sequence Current and Return Path](image)

Engineers often say that a “ground connection” is required for zero-sequence current to flow. Zero-sequence current would also flow in an ungrounded (isolated) neutral, but not grounding the neutral is very unusual for safety reasons. In this context, the terms neutral and ground are often incorrectly used interchangeably. Keep in mind that with a multiple-point grounded neutral, as is common on utility systems, the neutral conductor is electrically in parallel with the earth, so the zero-sequence current is returned by both the neutral conductor and the earth.

The circuit depicted in Fig. 10(b) becomes interesting when trying to apply a single-phase equivalent approach. The “single-phase” circuit is highlighted in Fig. 11.

![Figure 11 – Single-Phase Equivalent for Zero-Sequence Current](image)

Note that since the neutral conductor returns not only the zero-sequence current from the phase that we are considering as our single-phase equivalent, but also the zero-sequence current from the other two phases. This situation causes a problem since the current supplied ($I_0$) and the current returned ($3I_0$) are different.

This problem can be fixed quite simply by applying algebra. For the single-phase equivalent to be valid, the correct voltage drop must be calculated for the neutral return path. If the series reactance in the return path is $X_N$, the voltage drop for the neutral return path is found using Ohm’s Law.

$$V = (3I_0) \times X_N$$  \hspace{1cm} (13)

Forcing the current in the neutral return path of the single-phase equivalent circuit to equal the current supplied by the single phase ($I_0$), the coefficient 3 must be removed from the current $I_0$. Simply discarding this coefficient would change the calculated voltage drop for the neutral return path, thus invalidating the single-phase equivalent circuit. But the calculated voltage drop remains correct if the coefficient is simply grouped with the term ($X_N$), as shown in (14).

$$V = I_0 \times (3X_N)$$  \hspace{1cm} (14)

This subtle algebraic change has a significant physical interpretation. Any impedance in the neutral return path is subjected to three times the zero-sequence current as is flowing in each of the phase conductors; therefore, to provide the proper voltage drop, any impedance in the neutral portion of the circuit must be tripled when modeling the circuit as sequence networks. And since zero-sequence current is the only current component that can flow in the neutral, this condition applies only to the zero-sequence network.

**B. Sequence Networks**

Now that the modeling of unbalanced currents as symmetrical components is understood, the concept of sequence networks must be introduced. When a current $I$ flows through an impedance $Z$, the current should be interpreted as the sum of three balanced components. For phase $A$,

$$I_A = I_0 + I_1 + I_2.$$  \hspace{1cm} (15)

Each component of current can experience a different effective value of impedance. This rather abstract concept must be accepted, although the underlying reasons are not easily understood. Although far from a perfect analogy, one might consider a current containing several harmonic components. Each harmonic component experiences a different resistance value when flowing through a wire because AC resistance is a function of frequency. The sequence currents $I_0$, $I_1$, and $I_2$ are all at the system fundamental frequency, so the analogy is not perfect, but like the harmonic currents, the symmetrical components can each experience a different impedance value in a given portion of a system. Ohm’s Law can therefore be stated for each sequence component:

$$V_0 = I_0 \times Z_0, \quad V_1 = I_1 \times Z_1, \quad V_2 = I_2 \times Z_2.$$  \hspace{1cm} (16-18)

Since each component of current experiences a potentially different impedance, three different impedance networks must be developed for any system to be analyzed. Since most studies of unbalanced systems involve short-circuit fault calculations, neglecting the resistive portion of the impedance is a common practice, since its effect on the short circuit current magnitude is very small. For that reason, we will proceed to develop a positive-, a negative-, and a zero-sequence reactance network. Consider the one-line diagram shown in Fig. 12.

The positive sequence reactance network is developed directly from the one-line diagram of the system. First, we draw a Positive-Sequence Reference Bus. By convention, this bus is drawn at the top of the diagram. Although merely a convention, being consistent with this practice will facilitate both the proper network topology and the correct interconnection of the networks when the fault calculation is
done. After the Positive-Sequence Reference Bus is drawn, all sources and loads on the one-line diagram capable of storing energy (fault current contributors) are connected to it. Examples of fault contributors include utility system interconnections, generators, motors, and shunt capacitors.

The source impedances are modeled in series with an EMF source representing the pre-fault voltage at that point in the system. Since this voltage is generally not known unless a powerflow calculation is performed, it is often assumed as 1.0 per unit, and is assigned the reference angle of zero degrees. Next, the other components from the one-line diagram are modeled as reactances. Transformers T1 and T2 are drawn, and the location of Bus 1 is established. Finally, Transformer T3 and the location of Bus 2 are established, completing the positive-sequence network as shown in Fig. 13. Note that each numeric reactance value is the positive-sequence reactance for that component.

In the actual system, only positive-sequence voltages are generated. Therefore, all voltage sources will appear in the positive-sequence network only. Note that the reactances behind the generated voltages still appear in the negative-sequence network as negative-sequence reactances.

The negative sequence network for the one-line diagram shown in Fig. 12 is shown in Fig. 14.

The zero-sequence network can be developed directly from the negative-sequence network following these modification steps:

1. Relabel the reference bus as the “Zero-Sequence Reference Bus”
2. Change the numeric values of the reactances from the negative-sequence values to the zero-sequence values
3. Add three times the grounding impedance to the numeric reactance value of any machine that is grounded through an impedance
4. Adjust the topology of the network to force proper zero-sequence current behavior

The zero- and zero-sequence reactance values are substantially different for most components. A notable exception is the two-winding power transformer, where all three sequence reactances equal the leakage reactance of the transformer. Transmission and distribution lines have zero-sequence reactances that are higher than their positive- and negative-sequence reactances because of the impedance of the earth current return path. The earth’s resistance is determined using Carson’s Equations. Rotating machines, on the other hand, have zero-sequence reactances that are much lower than their positive- and negative-sequence reactances, due to the large in-phase magnitude of the zero-sequence flux across the air gap.

Step three above states the need to increase the numeric reactance value of any machine that is grounded through an impedance by three times the value of the grounding impedance. The grounding impedance is included only in the zero-sequence network because only zero-sequence current can flow to ground. The reason for tripling the grounding impedance is explained by the derivation of (14).

The fourth step above is to adjust the topology of the zero-sequence network to force the current flowing in that network...
to behave like zero-sequence current. Since zero-sequence current can only flow in the parts of a circuit that have a fourth conductor to serve as a return path, the delta and ungrounded wye portions of the system will not allow zero-sequence current to flow. The network topology must be altered by introducing open circuits and short circuits to the reference bus to reflect this fact.

These alterations are best understood by example. Before attempting an example, analyzing the zero-sequence current behavior of the delta-wye transformer is helpful.

Begin analyzing the delta-wye transformer shown in Fig. 15 with the wye circuit. In order for zero-sequence current to flow to the load on the phase conductors, the total zero-sequence current furnished (3 I₀) must return on the neutral.

![Figure 15 – Delta-Wye Transformer Zero-Sequence Current Behavior](image)

The per-unit zero-sequence current flowing in each of the wye-connected transformer windings must also flow in the corresponding delta-connected windings. Writing the node equation at each corner of the delta shows that no zero-sequence current can flow out of the delta onto the lines. The zero-sequence current flows in the wye-connected windings, and circulates in the delta-connected windings.

Note that if the circulating current in the delta is in the form of a third harmonic current, the resulting temperature rise due to the higher frequency (and skin effect) may be problematic. Third harmonic currents behave like zero-sequence currents.

In the zero-sequence reactance diagram, the zero-sequence current must be blocked from exiting the delta and flowing onto the lines of the three-wire circuit by introducing an open circuit on the delta side of the transformer. But that open circuit would also prevent zero-sequence current from flowing through the transformer impedance (windings), and we can see in Fig. 16 that this modeling is incorrect. So, a short circuit back to the reference bus allows the zero-sequence current to flow from the wye circuit, through the transformer reactance and to the reference bus, whereas the open circuit prevents the zero-sequence current from flowing out of the transformer to the delta circuit. This modification is consistent with Table I. Fig. 16 shows the zero-sequence circuit model for a delta-grounded wye transformer.

Using the alteration rules summarized in Table I, the topology of each machine (transformer, generator, and motor) reactance can be altered to allow proper zero-sequence behavior.

<table>
<thead>
<tr>
<th>Connection</th>
<th>Alteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grounded Wye</td>
<td>None</td>
</tr>
<tr>
<td>Wye</td>
<td>Open Circuit</td>
</tr>
<tr>
<td>Delta</td>
<td>Open Circuit AND Short Circuit to Ref. Bus</td>
</tr>
</tbody>
</table>

Table I – Zero-Sequence Network Alteration Rules

Note that the connection types of the motors shown in Fig. 12 are not specified. This missing information is not a problem, because motors are virtually always wired as a three-wire connection (either delta or wye – not grounded wye). According to Table I, both the delta and wye connections involve an open circuit, and this open circuit effectively removes the motor from the zero-sequence network.

![Figure 16 – Delta-Wye Transformer Zero-Sequence Circuit Model](image)

Begin developing the zero-sequence network from the negative-sequence network by relabeling the reference bus and changing the numeric reactance values from the negative-sequence values to the zero-sequence values. Next, transfer the phasing symbols (deltas, wyes, and grounded wyes) from the one-line diagram to the zero-sequence reactance diagram, as shown in Fig. 17(a).

At this point, any impedance grounded devices, such as the generator, must have their zero-sequence reactance increased by three times the grounding impedance. Fig. 17(a) shows the impedance of the generator increased from its original value of G to G + 3 Xn.

![Figure 17 – (a) First Step of Zero-Sequence Reactance Network Development and (b) Completed Zero-Sequence Network](image)

Finally, the topology alteration rules of Table I can be applied. Any grounded-wye circuit is left unaltered, since zero-sequence current can flow in a four-wire circuit. Ungrounded wye devices are open-circuited, because the lack of return path will prevent zero-sequence current from flowing in a three-wire circuit. And delta-connected components are altered to include both an open circuit (to prevent zero-sequence current from flowing on the three-wire circuit) and a short circuit to the reference bus (to simulate the circulating path provided by the delta-connected windings). The resulting zero-sequence network is shown in Fig. 17(b).

IV. ASSESSMENT OF EFFECTIVENESS

The approach for understanding symmetrical components and sequence networks of three-phase power systems as
described in this manuscript was taught in four different environments, and its effectiveness in each was assessed. The first environment was a three-credit power system analysis course (two trials), the second was a three-day short course involving 20 contact hours (three trials), the third was a one-day short course (one trial), and the fourth was a three-hour module of a professional engineering exam review seminar (four trials). The assessment method for all environments was participant feedback. The three-credit course also included assessment of test problem response as an evaluation method. The accuracy of test problem answers related to symmetrical components and sequence network synthesis for both undergraduates and graduate students were compared before and after implementing the teaching method described in this manuscript. A total of 56 students were assessed in the two after-implementation trials. These results were compared to the previous three sections of the course involving 61 students. An improvement was shown in each of the four categories of test problems. The problem topics requiring a mastery of the theoretical concepts showed more significant improvement, particularly the ability to properly synthesize a zero-sequence network from a one-line diagram.

<table>
<thead>
<tr>
<th>Problem Topic</th>
<th>Avg. score (before)</th>
<th>Avg. score (after)</th>
</tr>
</thead>
<tbody>
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<td>symmetrical components</td>
<td></td>
<td></td>
</tr>
<tr>
<td>theory</td>
<td>76%</td>
<td>88%</td>
</tr>
<tr>
<td>symmetrical components</td>
<td></td>
<td></td>
</tr>
<tr>
<td>application</td>
<td>82%</td>
<td>84%</td>
</tr>
<tr>
<td>sequence network</td>
<td></td>
<td></td>
</tr>
<tr>
<td>synthesis</td>
<td>66%</td>
<td>82%</td>
</tr>
<tr>
<td>zero-sequence network</td>
<td></td>
<td></td>
</tr>
<tr>
<td>synthesis</td>
<td>46%</td>
<td>78%</td>
</tr>
</tbody>
</table>

Table II – Results of Test Problem Answer Accuracy Before and After Implementation of the Novel Teaching Method

The short courses and review seminars were assessed by participant feedback, since these environments did not involve written tests. None of the participant feedback portrayed this novel teaching method in a negative light. Typical adjectives used to describe the method in which symmetrical components and sequence networks were presented included “clear,” “simple,” “straight-forward,” and “easy to understand.” In addition, close to 50% of the P.E. review students commented after the exam they were confident that they completed the fault analysis problems correctly, and cited the methods used in the review seminar as the reason for their success.

V. CONCLUSION

The method of symmetrical components is often shrouded in mystery, even among engineers who use it frequently. This phenomenon is usually a result of forcing an engineering problem into a mathematical equation without understanding the mechanisms underlying the mathematics. By drawing simple analogies to less abstract applications, a thorough understanding of the engineering problem can be attained. The process of synthesizing sequence networks is often problematic. An incorrectly developed zero-sequence network is likely to lead to an erroneous fault current calculation. The reason for most errors in the zero-sequence network stems from the method commonly used to synthesize the network. Most students are taught to apply zero-sequence circuit models for each component in the system. Using a “building block” approach, the entire zero-sequence network is built from the individual component circuit models. If done carefully – very carefully – this method will produce the correct network, but the chances of making an error are high. An alternate and less error-prone method of synthesizing the zero-sequence network, based on the understanding of the behavior of zero-sequence current, is presented in this manuscript.

It is the experience of this author that these techniques of mastering symmetrical components and sequence networks pay off in a much more thorough understanding of the topics. This observation is evidenced by greatly improved test scores among both undergraduate and graduate students. The students also demonstrate their level of comprehension by the mastery of more complex topics such as the behavior of harmonics. When a problem is thoroughly understood, a well-considered solution is a realistic expectation.

REFERENCES


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